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# Optimizing Diagnostic Effectiveness of Mixed Turbofans by Means of Adaptive Modelling and Choice of Appropriate Monitoring Parameters

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## ABSTRACT

Methods for the optimal selection of measurements and health parameters used for diagnostic purposes in aircraft gas turbine engines are presented. Principles of aerothermodynamic diagnostic techniques are first briefly reviewed. The problem of optimal selection of measurements and health parameters is examined from two different standpoints. (a) How to select out of all available measurements the minimum set that will be capable to provide sufficient information to assess engine health condition. (b) When a set of measurable quantities from an operating engine is given, how to select the combination of health parameters, in order to provide in an optimal way the information about the condition of the engine. The present paper concentrates mainly on the second type of problem since it is related to the handling of an existing fleet. Methods based on sensitivity analysis are discussed, but it is shown that the most substantial information is produced by analyzing the properties of the Jacobian matrix, interrelating parameters and measurement deviations. Finally, results of condition estimation for a number of turbofans in service are presented.

## Nomenclature

$a_{ij}$	Element of Jacobian matrix, eq (A9),(A10)
CN	Condition number of Jacobian matrix, eq(A14)
$EPR=P_7/P_2$	Engine Pressure Ratio
$F$	Prediction algorithm, eq(1)
$f$	(nx1) Vector of health parameters Eq. (13, 14)
$J$	Jacobian matrix
$m$	Number of measurements
$N$	Number of health parameters
$N_1$	Low pressure rotational speed
$N_2$	High pressure rotational speed
$P$	Total pressure along the gas path
$S_d$	Overall sensitivity operator, Eq (11)
$T$	Total temperature along the gas path
$u$	(3x1) Vector of variables defining the operating point, Eq. (1)
$V$	Matrix containing the singular vectors of $J$ Eq(A13)
$W$	Diagonal matrix containing the singular values of $J$ , Eq(A13)
$WFB$	Fuel Flow
$w_{\sqrt{g}}$	Corrected flow rate
$Y$	(mx1) Vector of measurable quantities, Eq(1,2)
$\Delta f$	Health parameter deviation from reference value, Eq(4)
$\Delta Y$	Measurement deviation from reference value, Eq(5)
$\eta$	Efficiency

## Superscript

ref Reference State

## Subscripts

actual	Actual value of parameter on component performance map
reference	Reference value of parameter on component performance map
s	Static Pressure or Temperature
2, 13, 23, 3, 41, 5, 7	Numbering of aerothermodynamic stations along the gas path, fig. 1

## 1. INTRODUCTION

The optimal exploitation of an existing fleet of engines is highly related with the ability to assess the health condition of each engine during operation or after overhaul. A large number of techniques for this purpose have been proposed in the past by several authors. A significant part of them uses aerothermodynamic measurement data in order to estimate the condition of each component and to assess the overall condition of the engine. Such techniques are characterized as gas path analysis techniques and are divided in two main groups.

The first group of techniques is referred to linear methods and comprises Least squares methods, Kalman filters and their derivatives. Health assessment is based on the formation of an appropriate matrix called Influence Coefficient Matrix, which relates deviations on measurable quantities from the reference state to deviations of health parameters under estimation. The fundamentals of Linear Gas Path Analysis have been posed by Urban (1972). Least Squares Techniques coupled with gas path analysis have been presented by Doel (1992, 1993). Techniques based on Kalman filters, have been presented by Provost (1994) and Volponi (1994), giving the possibility to distinguish between sensor fault and component fault. Stamatis and al. (1991) have proposed the linear gas path analysis in discrete operating points, a way to overcome the obstacle of limited available information from an operating engine.

The second group comprises the non-Linear methods. The estimation of the parameters describing the health condition of the engine is achieved using a non-linear model capable to simulate the engine behavior. These methods have been introduced by Stamatis and al. (1990a).

A common feature of both linear and non-linear techniques is the use of measurable quantities from an operating engine in order to estimate a set of diagnostic indices that reflect the health of the engine. Such indices can be the deviations in component performance parameter values, introduced by Urban (1972) or the component modification factors of Stamatis (1990a). Another common feature is the need of a performance model for the considered engine type. The derivation of health indices using linear or non-linear formulation can be achieved with the help of the model.

An issue of major importance is the selection of appropriate measurements and parameters, which will be used in the diagnostic procedure. Stamatis and al. (1992) have presented the method of sensitivity analysis for the optimal selection of measurements and health parameters with application to an industrial gas turbine. Provost (1994) has presented a method, which is capable to evaluate the correlation level among measurements or health parameters used on a diagnostic system. Tsalavoutas and al. (1999) have presented the implementation of the method based on sensitivity analysis for measurements and health parameters selection to the case of a twin spool, two-shaft gas turbine.

In the present paper measurement and parameter selection methods are presented, focusing on the case of a mixed, low by-pass ratio, turbofan. The implementation of the method presented in the past (Stamatis et al 1992) on this particular type of engine is examined, while some new ways of selection are introduced.

## 2. FUNDAMENTALS OF DIAGNOSTICS

For the purpose of the present analysis an engine is considered as a system, whose operating point is defined by means of a set of variables, denoted as  $u$ . The health condition of its components is assumed to be represented through the values of a set of appropriate "health parameters", denoted as  $f$ . The system is observed through measured variables, denoted as  $Y$ . In symbolic form we can write:

$$Y = F(u, f) \quad (1)$$

For a given operating point  $u$  the measurement values depend only on the health condition of engine components.

$$Y = F(f) \quad (2)$$

When an engine is in intact condition the health parameters have the reference values and the measurements have also reference values:

$$Y^{ref} = F(f^{ref}) \quad (3)$$

Occurrence of particular faults can be expressed through deviations of health parameters from the reference state. These deviations are defined for any health parameter as:

$$\Delta f_j = \frac{f_j - f_j^{ref}}{f_j^{ref}} \times 100 \quad (4)$$

Any mechanical or other alteration, which can be classified as engine fault, initially affects the performance of the damaged component and finally the overall performance of the engine, resulting to deviations on measured quantities:

$$\Delta Y_i = \frac{Y_i - Y_i^{ref}}{Y_i^{ref}} \times 100 \quad (5)$$

The relation between  $\Delta Y_i$  and  $\Delta f_j$  is, in general, non-linear. If deviations are small, however, a linear relation can be used as shown in APPENDIX 1, which can be written in a matrix form:

$$\begin{pmatrix} \Delta Y_1 \\ \Delta Y_2 \\ \vdots \\ \Delta Y_i \\ \vdots \\ \Delta Y_n \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{in} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_j \\ \vdots \\ \Delta f_n \end{pmatrix} \quad (6)$$

or in symbolic form:

$$\Delta Y = J \cdot \Delta f \quad (7)$$

where  $J$  is the Jacobian matrix.

The problem that has to be solved when an engine condition diagnosis is to be performed, is the determination of  $f$  when  $Y$  are known. In mathematical terms this means that eq. (2) has to be solved for  $f$  when  $Y$  is given:

$$f = F^{-1}(Y) \quad (8)$$

For this solution to be possible, it is essential that the number of equations must be equal to the number of unknowns, which translates to the fact that we must have as many different measured quantities as the number of unknown health indices. This is not sufficient however. To be able to solve the system of equations, this system must have a unique solution, namely the system representing the engine must be observable.

An equivalent approach is to determine  $\Delta f$  from  $\Delta Y$  if deviations are small. In this case eq. (7) gives:

$$\Delta f = J^{-1} \cdot \Delta Y \quad (9)$$

Of course derivation of  $f$  through eq. (8), or (9) would be reliable for noise free measurements. For actual measurements with noise, appropriate estimation techniques have to be used. The previous equations can nevertheless be used for the purposes of the present analysis, as they express the fundamental relation between measurements and health parameters.

Two questions can be stated at this point.

- For an engine represented through given set of health parameters, which is the best combination of measurements taken from an operating engine that is going to provide sufficient diagnostic information?
- For an available set of measurements taken from an operating engine, which is the best combination of health parameters to be estimated?

Methods for addressing these two questions are presented in the following.

### 3. MEASUREMENTS SELECTION

Information collected along the engine gas path determines the state of the working fluid inside the engine, reflecting any alteration, which happens to the engine. The amount of collected information depends on the number of sensors placed along the gas path. In order to have information for the condition of each component a large number of sensors would be necessary. These sensors should be placed in the inlet and the outlet sections of the components in order to have the full picture of the change that the components operation causes to working fluid. For example, in the case of a twin spool turbofan engine with 5 rotating components, a number of 12 sensors must be placed in the engine. That is not acceptable for an operating engine for several reasons: In general sensors and related systems are very expensive, increasing the complexity of the engine. Another important issue is that the addition of wall taps, pitot probe, and thermocouples causes disturbances on the flow. Also they are not always possible, because of harsh local conditions (for example measuring the TIT at combustion chamber outlet section). Not even on the phase of designing and testing a new engine such detailed methods are adopted.

It is thus desirable to have a minimum number of sensors placed on an engine. The choice of this minimum set has to be done without sacrificing diagnostic capability. In the following we will present methods, which allow selection of such measurements. It can be commented here that this part of the analysis is mainly addressed to engine manufacturers who want to select the instruments that must be placed on an engine under design, in order to ensure a good diagnostic ability in the engine. It can be also useful to the engine user, who wants to modify an existing instrumentation set.

#### 3.1 Sensitivity of measurements

A first means of examining which measurements are suitable for monitoring purposes is their sensitivity to component condition changes. It is shown by Stamatis et al (1991) that sensitivity of measurements can be used to select the most appropriate measurements to estimate a given set of health parameters. The sensitivity of measurement  $Y_i$  to health parameter  $f_j$  is defined as follows:

$$\Delta Y_i^j = \frac{Y_i^j - Y_i^{ref}}{Y_i^{ref}} \times 100 \quad (10)$$

when  $f_j$  deviates by a given amount from its reference value (typically 1%).

For a set of health parameters under estimation, the most appropriate measurements to be used are these, which exhibit the greater sensitivities on individual change of each parameter. Individual measurements are suitable for monitoring the condition of specific component. The way each measurement reflects the overall condition can be expressed through an overall sensitivity measure, defined as:

$$S\Delta Y_i = \left[ \frac{1}{n} \cdot \sum_{j=1}^n [\Delta Y_i^j]^2 \right]^{1/2} \quad (11)$$

Large overall sensitivity of a measurement indicates that at least one health parameter influences the examined measurement. The most sensitive measurements with respect to overall sensitivity can be selected in order to form the set to be used for a diagnostic system.

Although the most sensitive measurements should be included in the monitoring set, one additional condition has to be fulfilled: the measurements chosen have to be linearly independent with respect to parameters deviations. This means that the vectors of the influence coefficients for the measurements to be selected have to be linearly independent. A method to examine the degree of interdependence of measurements has been proposed by Provost (1994). Angles between the vectors are calculated and when found sufficiently large, they indicate a very small interdependence. The criterion proposed in the following subsection is another alternative method to select measurements, which are not linearly dependent.

### 3.2 Condition number of Jacobian. Examination of rows

The condition to have a unique solution is the sufficient observability of the diagnostic system. That issue is very common in control theory (Stengel 1993).

There is a well-known method to find out if a system is observable. This method is mainly related to the solution of linear system equations. When  $n$  health parameters have to be estimated and  $m$  measurements are possible ( $m > n$ ), we can select  $n$  out of these measurements to form a closed ( $n \times n$ ) system. The ones that will be chosen have to ensure (a) that that system has a solution (b) that the solution of the system gives the minimum amplification to the measurement noise. For a linear system of equations these conditions translate to the requirement for a small condition number for the matrix of the system. In APPENDIX 2 the relevant theorem and the principles of the method to determine matrix condition number are given. The criterion thus introduced is that the set of measurements that will be chosen will have to be such that the condition number of the system expressed by eq. (7) is the minimum possible. The procedure for materializing this criterion is described below.

We can form a number of square sub-matrices of the Jacobian. The number of these sub-matrices is given from the number of possible combinations of  $m$  available measurements taken  $n$  each time ( $n$  health parameters under estimation).

$$N_{comb} = \binom{m}{n} = \frac{m!}{n!(m-n)!} \quad (12)$$

Using the above described formulation we can investigate all the possible combinations of measurements for the given set of health parameters. The procedure to be followed is: (1) Using a performance model for the engine a  $m \times n$  Jacobian is formed, (2) All the possible square sub-matrices of Jacobian are formed, (3) The condition number of each sub-matrix is evaluated, (4) Results are sorted in ascending order of condition number.

At this point it must be said that the results must be judged from an engineering point of view as well. Common practices, which are in use, such as the desire to measure in the cold section of an engine, must be taken in to account, when the resulting combinations are inspected in order to select the most appropriate one.

## 4. HEALTH PARAMETERS SELECTION

A common feature of engines in service is the availability of minimum instrumentation, which accompanies the engine. This instrumentation is used primarily by the control of the engine, which ensures safe operation within its operating envelope. It is thus desirable to exploit the information provided from the sensors along the gas path in a way deriving the maximum diagnostic information about its state.

### 4.1 Singular Value Decomposition analysis of Jacobian matrix

In the general case we can form a Jacobian matrix, which has  $m$  rows corresponding to measurements and  $n$  columns ( $n > m$ ) corresponding to health parameters. Stamatis and al. (1991) have proposed the use of Singular Value Decomposition method in order to select  $m$  health parameters among  $n$  to form the most observable system for diagnosis of the state of the engine. According to that method the Jacobian matrix is decomposed as explained in APPENDIX 2.

Using the following lemma of linear algebra we can choose combinations of health parameters in order to form a well observable diagnostic system: *Linear combinations of elements of health parameters that correspond to the direction of the greater singular values vectors are estimated with the greater accuracy.* This implies that health parameters with the greater projections in the direction of the greater singular vectors are estimated with greater accuracy and must be preferred among all possible.

The procedure to be followed is: (1) Using a performance model for the engine considered, a  $m \times n$  Jacobian is formed, (2) Jacobian is decomposed using SVD analysis, (3) The singular values of Jacobian are sorted in descending order and the singular vectors are properly rearranged, (4) Results are inspected in order to select the best combination of parameters.

### 4.2 Condition number of Jacobian. Examination of columns

The approach presented above for measurement selection using the condition number of Jacobian can also be used for parameter selection. In this case sub-matrices are formed by combining columns, which correspond to health parameters.

1. Using a performance model for the engine type considered, a  $m \times n$  Jacobian is formed.
2. All possible square sub-matrices of Jacobian are formed.
3. Each one of the sub-matrices is decomposed using SDV analysis and the condition number is evaluated.
4. Results are sorted in ascending order of condition number.

The combinations, which exhibit the smaller condition numbers, are more suitable to be estimated from the diagnostic system.

Engineering judgment should also be applied. Since the problem is not just mathematical but also physical, the chosen combination must supply the diagnostic procedure with substantial information. For example, existing experience may indicate that certain parts of the engine are more prone to damage than others. This means that some health indices should be included in the set to be defined while some others could be kept constant. This means that the combination to be selected, must rank high in the list (small condition number) but should not necessarily be the first.

## 5. APPLICATION OF PRESENTED METHODS

Application of the methods presented above is demonstrated for a twin spool mixed turbofan engine. The layout of the engine is shown in figure 1.

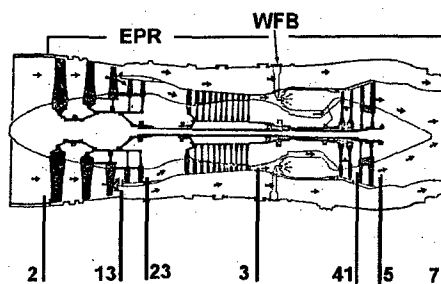


Figure 1: Layout of twin spool mixed turbofan

The data for the particular engine simulated are chosen to represent the Allison TF-41 engine. This particular engine was chosen because test cell test data were available, and are used later in the paper.

The model created simulates engine behavior for a wide range of operating conditions. The simulation is done using the operating point definition variables and the health parameters. The operating point  $u$  is defined using the desired value for net thrust and the total conditions  $P_2$ ,  $T_2$ , at fan inlet section (station 2). A diagnostic ability is incorporated in the model through the use of appropriate component health parameters. In the present study the considered health parameters are these introduced by Stamatis (1989). For each rotating component, two health parameters are defined:

$$\text{Flow capacity: } f_j = \left( \frac{W\sqrt{\vartheta}}{\delta} \right)_{\text{actual}} / \left( \frac{W\sqrt{\vartheta}}{\delta} \right)_{\text{reference}} \quad j=1,3,5,7,9 \quad (13)$$

$$\text{Efficiency: } f_j = \eta_{\text{actual}} / \eta_{\text{reference}} \quad j=2,4,6,8,10 \quad (14)$$

The set of parameters is shown in Table 1.

Table 1: Health parameters for the engine components

Considered Health Parameters for mixed turbofan		
Symbol	Description	
1	$f_1$	Flow capacity factor at FAN
2	$f_2$	Efficiency factor at FAN
3	$f_3$	Flow capacity factor at IPC
4	$f_4$	Efficiency factor at IPC
5	$f_5$	Flow capacity factor at HPC
6	$f_6$	Efficiency factor at HPC
7	$f_7$	Flow capacity factor at HPT
8	$f_8$	Efficiency factor at HPT
9	$f_9$	Flow capacity factor at LPT
10	$f_{10}$	Efficiency factor at LPT

### 5.1 Measurements Selection

The objective is to select ten measurements in order to estimate ten health parameters. For that set a sensitivity analysis has been performed. Each health parameter has been disturbed by -1% from reference and measurement sensitivity is derived. The procedure is repeated for all health parameters under consideration. In figure 2 the overall sensitivity of each measurement to all health parameters is shown.

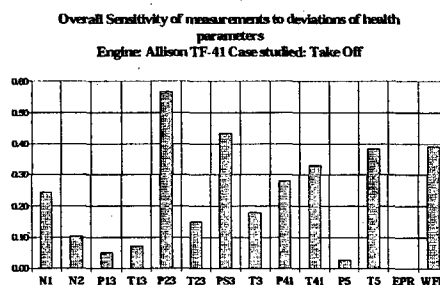


Figure 2: Measurements overall sensitivity

For the examined case we can easily see that the most sensitive measurements in an order of descending overall sensitivity are:  $P_{23}$ ,  $P_{33}$ ,  $WFB$ ,  $T_5$ ,  $T_{41}$ ,  $P_{41}$ ,  $N_1$ ,  $T_3$ ,  $T_{23}$  and  $N_2$ . With the use of these ten measurements we can proceed to the estimation of the ten considered health parameters. In this point we must say that if two measurements exhibit large sensitivity but they are also correlated between them, then the one must be removed from the diagnostic set.

It is worth observing that *EPR* exhibits practically zero sensitivity. This is an immediate consequence to the fact that *EPR* is related to thrust in an almost one-to-one relationship, irrespective of components condition. Since deviations were calculated for a constant thrust, *EPR* practically did not change either.  $P_5$  is a quantity with a value very close to the value of  $P_7$ , which sets the *EPR*. That's why it shows a very small sensitivity too.

In order to have a more precise image for the sensitivity of each measurement we can inspect the result of measurement sensitivities to individual changes of health parameters. In figure 3 the sensitivity of measurements to deviation of factor  $f_1$  (flow capacity of Fan) is shown. Inspection of the figure shows that some measurements exhibit a large degree of sensitivity in comparison with others. These measurements should be included in order to form a diagnostic system for the estimation of  $f_1$ . For the case studied ( $\Delta f_1 = 1\%$ ) the most sensitive measurements are  $P_{23}$ ,  $N_1$ ,  $P_{41}$ ,  $P_{53}$ ,  $T_{23}$  and *WFB*. Measurements like  $P_5$ ,  $T_{41}$ , which are insensitive to change of  $f_1$ , are not suitable for the estimation of that parameter.

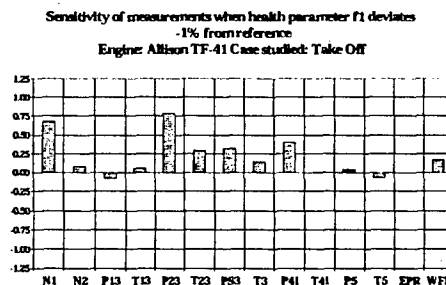


Figure 3: Measurements sensitivity to deviation of  $f_1$  from reference

We proceed now by applying the method based on the condition number of the linear system. A  $14 \times 10$  Jacobian has been generated and 1001 square sub-Jacobians are examined. The condition number of each one is derived and results are sorted in ascending order. Figure 4 shows the Condition Number values for all combinations. There is a fast rise of condition number in the first hundred combinations. This rise of condition number corresponds to a progressive degradation of the quality of the examined combinations of measurements for the given set of health parameters. In Figure 5 we can see in detail the rise of condition number and also the first fifteen combinations. It is worth noting that *EPR* is not contained in the first fifteen combinations. Actually, the first combination containing *EPR* has a condition number about two orders of magnitude larger than the smallest one.

At this point it should be mentioned that the method of Provost (1994) has also been applied to the present data. Angles between the measurement vectors were calculated in all possible combinations. The measurement pairs whose vectors were found to form small angles were  $P_5$ ,  $P_{13}$  and  $T_5, T_{41}$ . It should therefore be desirable to avoid having both measurement of each one of those pairs in a measurement set. This observation is in agreement with the analysis using the condition number, as combinations containing those pairs are only found for set with higher condition numbers.

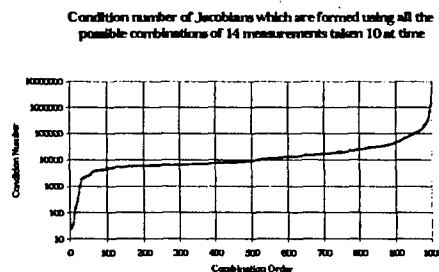
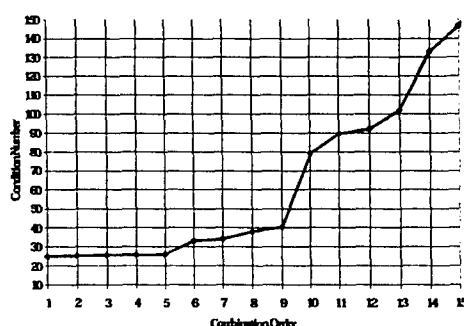


Figure 4: Condition Number of measurements sets in ascending order \_



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$N_1$															
$N_2$															
$P_{13}$															
$T_{13}$															
$P_{23}$															
$T_{23}$															
$P_{53}$															
$T_3$															
$P_{41}$															
$T_{41}$															
$P_5$															
$T_5$															
<i>EPR</i>															
<i>WFB</i>															

Figure 5: The lowest condition numbers and corresponding measurement sets

## 5.2 Health parameters Selection

In the actual TF-41 engine installed in a test cell seven measurements are available:  $N_1$ ,  $N_2$ ,  $P_{S3}$ ,  $T_3$ ,  $T_5$ ,  $P_5$  and  $WFB$ . It is obvious that the estimation of the full set of health parameters is not possible and a limited set of those must be chosen. Thus we must choose seven health parameters in order to form a square diagnostic system.

First the 7x10 Jacobian matrix is formed. The 7x10 Jacobian is given on table 2. Operating point is set defining the total inlet conditions  $P_2$ ,  $T_2$  for a given level of thrust. For these settings following the procedure described at APPENDIX 1, the Jacobian obtained is shown in table 2.

Table 2: Jacobian matrix for 7 measurements and 10 parameters

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$	$f_8$	$f_9$	$f_{10}$
$N_1$	-0.68	0.18	0.00	0.06	0.05	0.13	-0.05	0.16	-0.07	0.21
$N_2$	-0.08	-0.13	0.03	-0.06	-0.18	0.11	-0.08	0.13	0.10	0.00
$P_{S3}$	-0.32	-0.05	0.33	-0.04	0.04	0.14	-1.23	0.21	0.29	-0.12
$T_3$	-0.14	-0.24	0.04	-0.12	-0.10	-0.25	-0.30	0.18	0.17	-0.03
$T_5$	-0.04	-0.01	0.04	-0.01	0.00	0.01	0.00	0.01	0.04	-0.04
$P_5$	0.07	-0.49	-0.27	-0.16	-0.21	-0.54	0.22	-0.67	-0.02	-0.55
$WFB$	-0.16	-0.61	-0.05	-0.18	-0.18	-0.46	0.19	-0.57	0.19	-0.68

Before proceeding to selecting parameters we observe that the elements on the row that is referred to  $P_5$ , have very small magnitude in comparison to the elements of the other rows. This means that the measurement  $P_5$  is insensitive to deviations of all considered parameters under estimation and the presence of  $P_5$  on the diagnostic system do not supply us with extra information to be used in the estimation. Inspection of figure 5 shows also that  $P_5$  is not contained simultaneously with the remaining six measurements, for any of the sets with low condition number. Thus another problem appears. If we decide to keep  $P_5$  then we introduce a singularity on the diagnostic system. If we decide to remove it then we should decrease the number of health parameters under estimation from seven to six.

We have applied the method of SVD analysis for health parameters selection for both cases and the projections of the singular vectors in the direction of the greatest Singular Value are given in figure 6. It is very clear that if we take in to account the matrix containing 7 rows ( $P_5$  is included) or the matrix containing 6 rows ( $P_5$  is removed), the results are almost exactly the same, which means that the inclusion of  $P_5$  in the diagnostic system does not offer significant information and for this reason the number of health parameters under estimation must be decreased.

Inspecting figure 6 we can perform a selection for health parameters using the criteria of section 4.1. Placing the projections of singular vectors in descending order with absolute norm we see that the first six projections correspond to the following health parameters:  $f_8$ ,  $f_{10}$ ,  $f_2$ ,  $f_6$ ,  $f_7$ ,  $f_3$ .

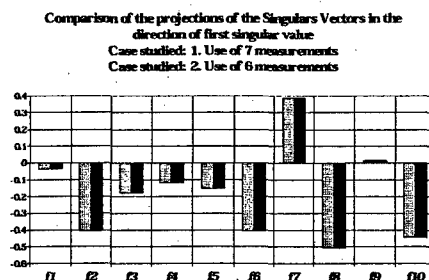


Figure 6: Projections of Singular vectors in the direction of first singular value

We come now to the selection of parameters using Jacobian condition number. Two cases have been examined and results have been derived. First a selection of seven health parameters is performed. Next  $P_5$  has been removed from the set of measurements and calculations for the selection of six health parameters are repeated.

Figure 7 shows the condition number of all square matrices, which can be formed using all possible combinations of 10 health parameters taken 7 at a time. Figure 8 give the lowest Condition numbers and corresponding measurement. The resulting combination of health parameters is  $f_1$ ,  $f_2$ ,  $f_3$ ,  $f_6$ ,  $f_7$ ,  $f_9$ ,  $f_{10}$ .

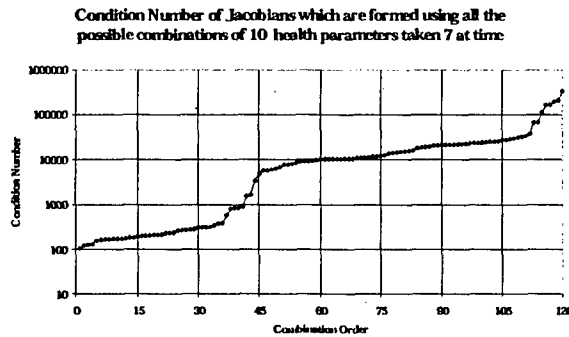


Figure 7: Ascending order of CN for parameters selection. Case of 7 available measurements

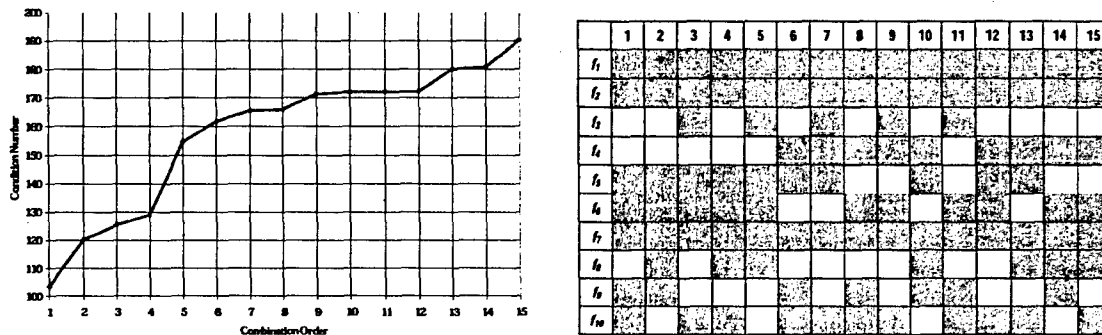


Figure 8: The lowest Condition numbers and corresponding measurement set. Case: 7 measurements and 10 parameters

As discussed above  $P_5$  can be removed from the diagnostic system because is insensitive in the changes of health parameters under estimation. In order to find out the influence that such a decision has on parameters estimation a study for the selection of six parameters has been performed. Figure 9 shows the condition number of square Jacobians formed using all possible combinations of ten health parameters taken six at time. A first observation is that the combinations appearing at the higher positions of the ascending order have condition numbers, which are one order of magnitude lower than the previous case. We can see now that the amelioration of diagnostic system with the removal of  $P_5$  has an immediate consequence on the condition number of Jacobians. Figure 10 gives the condition numbers for the first fifteen positions and also the actual combinations of parameters. For this studied case the best combination is:  $f_1, f_3, f_6, f_7, f_9, f_{10}$ .

The method of Provost has also been applied to the parameters and it was found that with the particular measurement set, the pair of parameters  $f_3, f_4$  have a high degree of correlation.

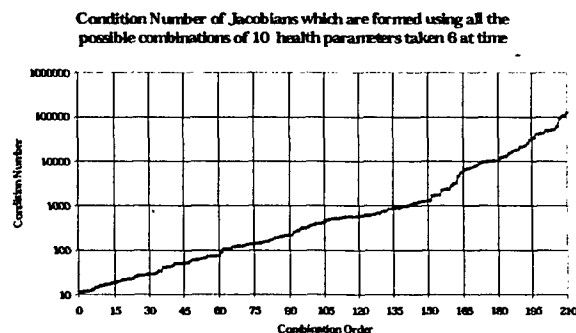


Figure 9: Ascending order of CN for parameters selection. Case of 6 available measurements

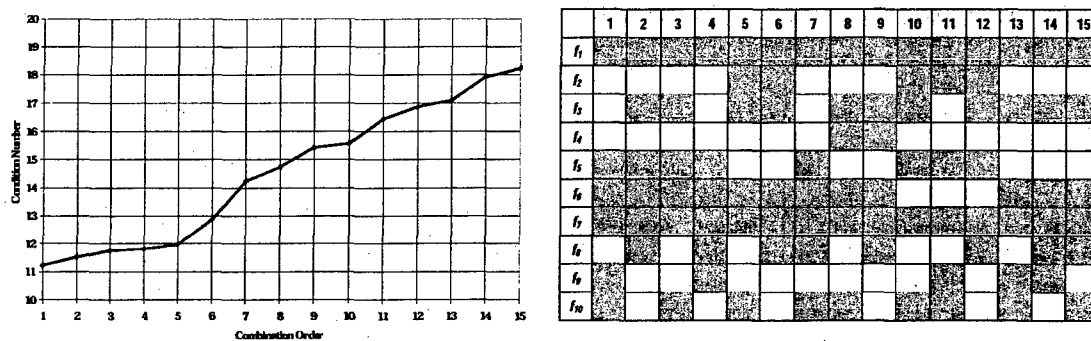


Figure 10: The lowest condition numbers and corresponding measurement set. Case: 6 measurements and 10 parameters

## 6. CONDITION ASSESSMENT OF A TURBOFAN, FROM TEST CELL DATA

The conclusions drawn from the parameter selection approach have served as a guide for pursuing a diagnostic study on the basis of a number of test reports from test cell runs of TF41 engines. The method of adaptive modeling (Stamatis et al 1990a) has been used.

First issue on the setup of the system is the adaptation of the general model to the performances of the considered engine type. This adaptation is related with the ability of the model to assess the health condition of the engine. When a model is adapted to the performance of a particular engine and a reference state has been established then it is ready to assess the health condition of the engine. The combination of health parameters that is used for adaptation and diagnosis is the sixth one of figure 10 namely  $f_1, f_2, f_3, f_6, f_7, f_8$ . We have selected that combination because it contains parameters expressing the full state (flow capacity and efficiency) of two components of the engine namely fan and HPT, which are more inclined to faults.

Test data were available from a "calibration" engine. These data have been used to calibrate the model, namely to produce the values of the health parameters that will be used as reference. For the data coming from this engine the adaptation procedure is performed and the results are given in figure 11. Each one of the runs of this figure corresponds to a different level of thrust.

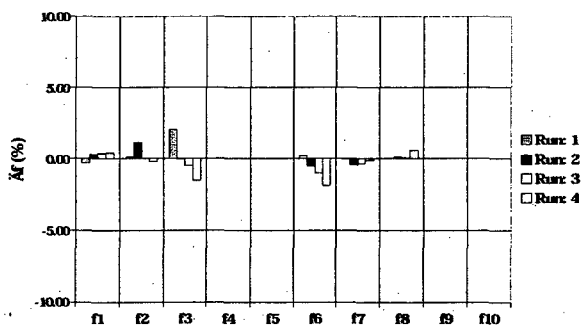


Figure 11: Results of adaptation procedure for the reference engine

Health parameters representing the reference state of TF-41 have very small values. Their values change with thrust level, a fact that is taken in to account in later exploitation of the model. These values are going to be considered as initial ones (reference) for the health parameters under estimation.

After establishing the reference state of TF-41 using the reference engine we have used a series of data coming from tests of different engines in order to see what the range of variation of the different parameters is.

In figure 12 results of applying adaptive modeling are presented. Results are presented for the six health parameters and are given for all examined engines without any classification according to engine or thrust level. For these reasons the results exhibit a large scattering. A close look at results can provide a better insight on the significance of the values produced by adaptive modeling.

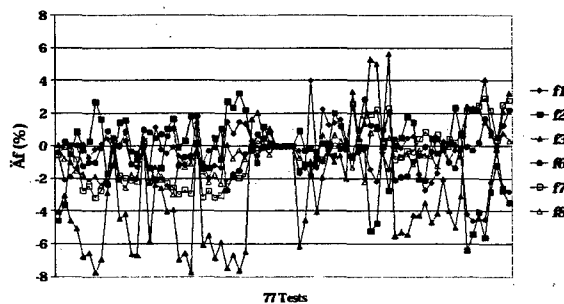


Figure 12: Results of diagnostic procedure from 77 tests

In figure 13 (a) and (b) results for the same engine before and after maintenance are shown. We see that the health parameter  $f_1, f_2, f_7, f_8$  come closer to the reference value after maintenance.

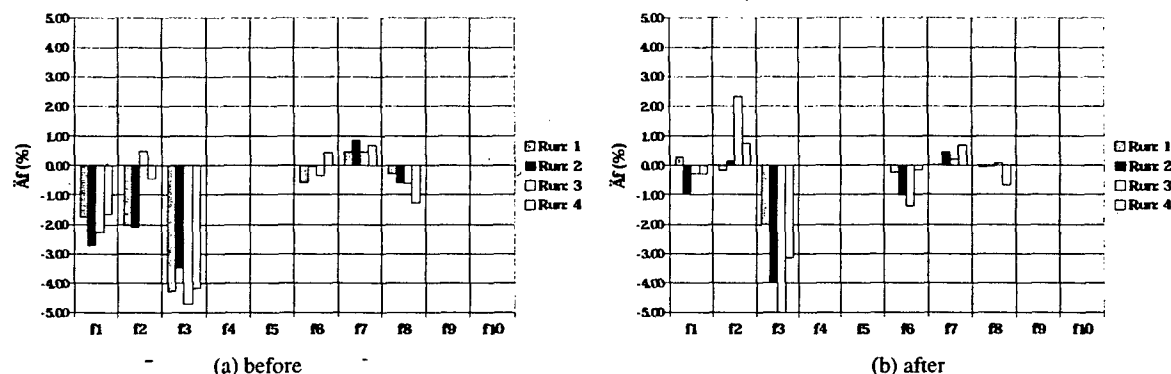


Figure 13: Results of diagnostic procedure before and after maintenance

An important issue, which is related with the handling of an existing fleet, is the degradation of operating engines with time. The health parameters employed by the present method give the possibility to assess the level of deterioration in individual engine components. We will show some indicative results supporting this point, coming from the data sets, which were available. Unfortunately no consistent data sets for one particular engine were available at successive instants in time. Nevertheless, data from tests of engine with different life spans were available. Health parameters from three different engines, each one having a different time operating hours, are shown in figures 14. Although engine-to-engine differences do not permit a strict comparison of values, it is evident that performances deteriorate as the 'age' of the engine increases.

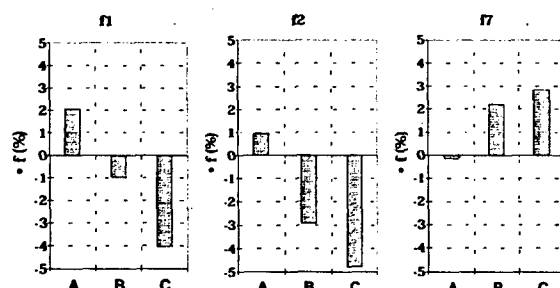


Figure 14: health parameters for three different engines  
(A: "New" Engine, B: 1200 hours of operation, C: 5000 hours of operation)

## 7. CONCLUSIONS

Methods for the optimal selection of measurements and health parameters for diagnostic purposes in aircraft gas turbines have been presented. First, method based on sensitivity analysis and singular values decomposition of the Jacobian, have been used. A method adding significant information was then introduced, based on the condition number of the Jacobian.

The method of indices selection was then applied to the case of a given turbofan, to select a set of parameters that can be estimated, to describe engine condition. These parameters were then evaluated from data sets coming from test cell runs.

It was found that if the overall set of engines is looked at, an apparent scattering of values exists, indicating that each individual engine has to be followed separately. It was shown that the effect of engine overhaul can be assessed using the chosen set and method.

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## APPENDIX 1: Linear and Non-Linear approaches of parameters estimation

Estimation of health parameters can be achieved using the inverse algorithm of  $F$ , eq (1).

$$f = F^{-1}(Y) \quad (A1)$$

Since in general faults are expressed with small changes of health parameters from the reference state, a simplification of above presented algorithm can be performed. When all considered health parameters are in the state of reference engine response will be:

$$Y^{ref} = F(f^{ref}) \quad (A2)$$

When a fault occurs some health parameters modify their values from reference with the following form.

$$f_j = f_j^{ref} + \delta f_j \quad (A3)$$

Where  $\delta f_j$  is a small quantity (magnitude in the range 0.01 to 0.03). The response of the engine to these changes will be:

$$Y = F(f) \quad (A4)$$

Since the changes  $\delta f_j$  are very small, using the Taylor's theorem for each  $Y_i$  we can write:

$$Y_i = Y_i^{ref} + \sum_{j=1}^n \frac{\partial F_i}{\partial f_j} \cdot (f_j - f_j^{ref}) + H.O.T. \quad (A5)$$

Where H.O.T. are the higher order terms that have a very small influence on the sum because the change  $f_j - f_j^{ref}$  is considered very small. We can thus write:

$$Y_i - Y_i^{ref} = \sum_{j=1}^n \frac{\partial F_i}{\partial f_j} \cdot (f_j - f_j^{ref}) \quad (A6)$$

and taking the percentage changes we can have:

$$\frac{Y_i - Y_i^{ref}}{Y_i^{ref}} = \sum_{j=1}^n \frac{\partial F_i}{\partial f_j} \cdot \frac{f_j^{ref}}{Y_i^{ref}} \cdot \frac{f_j - f_j^{ref}}{f_j^{ref}} \quad (A7)$$

- or

$$\Delta Y_i = \sum_{j=1}^n a_{ij} \cdot \Delta f_j \quad (A8)$$

$$\text{where } a_{ij} = \frac{\partial F_i}{\partial f_j} \cdot \frac{f_j^{ref}}{Y_i^{ref}} \quad (A9)$$

This relation is written in a matrix form as eq. (6) of the main text.

Under these conditions the non-linear model can be replaced by a linear relation between deviations on measurements and deviation on health parameters. Matrix  $J$  (Jacobian) contains the elements  $a_{ij}$  and can be created using a performance model. Each element is derived by the following formula:

$$a_{ij} \equiv \frac{\frac{Y_i^j - Y_i^{ref}}{Y_i^{ref}}}{\frac{f_j - f_j^{ref}}{f_j^{ref}}} = \frac{\Delta Y_i^j}{\Delta f_j} \quad (A10), \text{ where: } Y_i^{ref} = F_i(f^{ref}) \text{ and } Y_i^j = F_i(f_1^{ref}, f_2^{ref}, \dots, f_j, \dots, f_n^{ref})$$

or in symbolic form:

$$Y^j = F(f^{ref} + e_k \cdot \delta f_j) \quad (A11)$$

where  $\delta f_j$  small quantity and

$$\begin{cases} e_k = 0, \text{ for } k \neq j \\ e_k = 1, \text{ for } k = j \end{cases} \quad (A12)$$

**APPENDIX 2: Condition number of matrices.**

A norm of matrix condition is the condition number. In the open literature there are several definitions about the condition number of a matrix. A very convenient definition for the purpose of the present task is the one using the singular values of the matrix under consideration (Press et al 1992). We have based our effort to that definition since it is very easy and quick to be performed and also it is capable to demonstrate the level of linear independence between the rows or the columns of the matrix. The method consists in applying singular values decomposition of the matrix, using the well-known theorem in the following form (Lancaster et al 1985):

$$J = U \cdot W \cdot V^T \quad (A13)$$

Where  $W$  is diagonal containing the singular values of matrix  $J$  and  $V$  is the matrix whose columns contain the singular vectors of  $J$ . Each column of matrix  $V$  can be considered as a separate vector that corresponds to each derived singular value. The coordinates of these vectors are referred to each health parameter.

The condition number of matrix  $J$  is defined as the ratio of greatest singular value to the smallest.

$$CN = \frac{\max\{W_j\}}{\min\{W_j\}} \quad (A14)$$

It is known that well-conditioned matrices will have small condition numbers. As condition number increases, matrix condition deteriorates.

## Paper 9: Discussion

### Question from Dr R Szczepanik – Instytut Techniczny Wojsk Lotniczych, Poland

According to your experience and presented methodology, which parameters can be defined as the most effective and valuable for fault diagnostics and health monitoring purposes?

#### Presenter's Reply

For the particular engine we analysed (TF41), the most effective parameters are the ones given in the paper. For another engine type, the methodology would have to be applied in order to generate a specific answer.

### Question from W P J Visser – NLR, Netherlands

Is the methodology equally applicable to non-mixing turbofans, and is N1 suitable, instead of EPR, as a thrust setting parameter for the reference operating point?

#### Presenter's Reply

The methodology is applicable to any gas turbine engine, provided there is a performance model available; so a non-mixing turbofan can also be analysed.

For the particular engine we considered, we found that EPR has an advantage over N1 as a thrust setting parameter, in that its correlation with thrust is influenced less by changes in engine condition caused by faults and deterioration.

### Question from H G Cook

You have discussed the optimisation of measurements and parameters but, traditionally, some of these are not measured on the installed engine. Can current/traditional measurements be optimised to the same effectiveness or is there a case, possibly in life cycle cost, to move to the use of new parameters/measurements?

#### Presenter's Reply

The measurements for the test case presented in the paper come from a test cell and are more comprehensive than those available on an installed engine. The procedure presented, however, is generic and can be applied to the measurement set of an installed engine. Effectiveness will be reduced in the case of fewer measurements. The optimisation can, nevertheless, provide new parameters that will improve effectiveness over current practice.